Mathematical Discourse that Breaks Barriers and Creates Space for Marginalized Learners

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ROBERTA HUNTER AND MARTA CIVIL

INTRODUCTION

This book is the result of work that started at the annual meetings of the Psychology of Mathematics Education (PME) in 2014 and 2015. In 2014 we organized a Discussion Group on the general topic of this book (Civil, Herbel-Eisenmann, Hunter, & Wagner, 2014), which was followed by a Working Session in 2015 (Civil, Hunter, & Planas, 2015). Through the discussion and working sessions we engaged the participants in considering ways in which marginalized students could be provided equitable access to mathematical discourse and practices. In particular, we looked at two different tools to analyze a video clip of a group of students of Mexican origin in the US as they engaged in problem solving in their home language (Spanish) (Civil, 2012). One of the tools was a Communication and Participation Framework (Hunter & Anthony, 2011) designed to support teachers to scaffold diverse students to engage in mathematical practices. The other tool centered on authority structures within the classroom (Herbel-Eisenmann & Wagner, 2010; Wagner & Herbel-Eisenmann, 2014). While these two tools were quite different they shared a common interest on the participation of marginalized students in the mathematical discourse. This work served as catalyst for the book we present here.

Participants of the 2014 Discussion Group or the 2015 Working Session were invited to contribute a chapter that explored how barriers to the discourse have been identified and spaces created for different groups of minoritized students. We aimed for a representation across a range of countries.

Our aim for this book is to illustrate the different ways in which marginalized learners are provided with space to equitably access discourse in the mathematics classroom. The various chapters tell the story of equitable practices for diverse learners within a range of different contexts. By diverse learners we mean learners who are minoritized in terms of gender, disability, or/and their social, cultural, ethnic, racial or language backgrounds. In this book we aim to increase our understanding about what it means to imagine, design and engage with policy and practice which enhance opportunities for all students to participate in productive mathematical discourse. We aim to explore a widened lens on the contexts in which mathematical learning with minoritized learners is either promoted or occurs.

In widening the lens across policy and practice settings we recognize the interplay between the many complex factors that influence student participation in mathematics, and the learning of mathematics, as learners engage in mathematical discourse. Thus each chapter illustrates a variety of research perspectives, empirical traditions, and conceptual foci. Different aspects of diversity are raised, and at times conventional wisdom is challenged as the authors provide insights as to how educators may address issues of equitable access of minoritized learners to the mathematical discourse within settings across early primary through to high school, and situated in schools or in family and community settings. The intent of the book is to support mathematics education researchers, mathematics teacher educators and policy makers in providing positive solutions to the enduring challenge in mathematics education of enabling *all* participants including minoritized students to equitably access the mathematical discourse.

The chapters in the book have been separated into two sections. The first set of chapters provides insight into the ways participation of minoritized students in mathematical discourse has been supported in classroom, school and district settings.

Within Chapters One, Two, and Three insights are provided into the positive actions teachers take which support the construction of learning environments which enable all students to access the mathematics, the mathematical practices and the social interactions which support them gaining deeper understandings. While Chapter One provides a smart tool designed to support teachers to provide opportunities for students to access a range of mathematical practices within reasoned discourse, Chapter Two presents two theoretical lenses. These lenses enable identification of equitable patterns of interactions for all learners. At the same time the chapter provides a range of practical strategies which teachers can consider as ways to disrupt inequitable patterns linked to constructed status hierarchies within not only small group activity but also across whole class discussions. Chapter Three focuses attention on students who have sound everyday language but who may struggle with the linguistic means to participate in providing mathematical explanations. Teacher moves are identified which support and foster a classroom climate which provides space to enable all students to participate.

Chapter Four continues to direct our attention to the need to consider diverse ways of participation with the provision of two frameworks to explore, that of silence and precision. In this chapter and the next one we are reminded that language is one of several resources and that participation can include silence and so instruction needs to extend its gaze beyond talk to include posture, gaze and gesture. Within Chapter Five the theoretical lens of 'a pedagogy of possibility' which affirms student proficiencies across different languages supports notions of bilingualism as an enhancer.

Chapter Six shifts attention to the role formative assessment can hold as a tool to reduce marginalization of students in the discourse. Continuous formative assessment, responsive teaching and utilization of effective patterns of variation are critical features.

The next chapter in this section takes us beyond classrooms and into a district of schools where over a three year journey the goal was to make student discourse the norm across classrooms. Within Chapter Seven levers are identified that make district change possible while at the same time insights are provided of the complexities of putting theory into practice.

In Chapter Eight the focus is placed on what is termed ableism perspectives. Within these perspectives the argument is made that the mathematical agency of learners with disabilities is either supported or restricted by how their contributions are valued, managed and attuned to. Through supporting teachers to examine tasks for the potency of learners' mathematics proficiency and their use across a whole diverse class, these authors suggest that this may result in resignification in contrast to 'othering' within the learning environment. In Chapter Nine we are taken into the mathematical world of deaf students where the use of sign language is intertwined within the social, semiotic and individual discourse. Clear classroom illustrations are provided of how discourse and social interactions are shaped differently through signs and gestures.

The chapters in the second section of the book are positioned within diverse cultural and language spaces in different parts of the world. The connecting link across the chapters is that they are all set within indigenous communities. In Chapter Ten three cultural models are provided and the argument is made that unless teachers understand the lived world of the indigenous student the nature of mathematical activity alone is not sufficient to ensure discourse enabling or culturally sustaining practice. Chapter Eleven also argues the need for teachers to examine their own cultural values at the cultural interface between two knowledge systems (the indigenous and the Western World) as a prerequisite for having cultural awareness in the world of the students.

Chapters Twelve and Thirteen maintain a focus on culture and the significance of students interacting within the mathematical discourse using their home language but extend across schools, and into professional development settings with teachers. In Chapter Twelve we are provided with a picture of successful cultural practices across a group of remote Aboriginal schools that shaped the interactions in the mathematics classroom. A key element included community members co-teaching with the mathematics teachers so that the strategies drew on home cultural practices. In Chapter Thirteen we see the focus continue to be placed on the need for teachers to identify and respond in appropriate ways to the cultural and linguistic strengths of the students. Descriptions are provided of how through teacher workshops the teachers were supported to design culturally responsive mathematical activity.

The final chapter introduces issues and challenges around students who are multilingual in mathematics classrooms. The authors present three cases across three diverse continents as a way to explore spaces for the participation of all learners in the mathematical discourse. They present a thought provoking chapter to conclude the book with, in their identification of not only opportunities students are provided with in the mathematical discourse but also barriers which may emerge linked to official/national-home-world trilingualism in the mathematics classroom.

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ROBERTA HUNTER AND JODIE HUNTER

1. OPENING THE SPACE FOR ALL STUDENTS TO ENGAGE IN MATHEMATICAL PRACTICES WITHIN COLLABORATIVE INQUIRY AND ARGUMENTATION

INTRODUCTION

In recent times reform efforts have placed importance on student engagement in collaborative mathematical discourse as an essential component for their learning of mathematics with understanding. Central to these reform changes is a shift towards considering “mathematics classrooms as learning communities rather than merely a collection of individuals” (Hodge, 2008, p. 32); a shift commonly supported in many Western policy statements. For example, in the most recent New Zealand curriculum document focus is placed on students being able to communicate reasoning and “listen actively, recognise different points of view, negotiate and share ideas” (Ministry of Education, 2007, p. 12). In this document teachers are directed to establish learning environments that provide students with access to learning partnerships, challenging conversations, and responsive feedback. In this document, however, teachers are not provided with any guidance on how to support students to engage in mathematical dialogue premised within inquiry and argumentation. There is, also, little consideration given to student diversity within classroom communities. In classrooms where there is increased mathematical talk, only some students may be privileged while others are marginalized. Our aim in this chapter is to grapple with issues of equity within inquiry classrooms where student engagement in high levels of mathematical discourse are emphasised. We will show how equitable outcomes can be achieved when teachers open the space and explicitly reposition all students to access the mathematical discourse.

Many teachers are pedagogically challenged when required to enact classroom practices where student reasoning is the focus of mathematical discussion. A number of researchers (e.g., Hodge, 2008; McCrone, 2005; Sherin, 2002; Wood, 1999) outline how constructing inquiry classrooms which use the discourse of student explanation and which extend to inquiry, justification, and generalisation of mathematical thinking increases the complexity of teaching. Shifts in the interactive patterns within the learning environment require teachers to change traditional patterns of teacher-led talk to a more open and often student-promoted dialogue in response to a range of mathematical contributions from members of the community. For teachers this requires them to adopt more facilitative roles in the discussion

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and more adaptive forms of pedagogy (Franke, Kazemi, & Battey, 2007; Mendez, Sherin, & Louis, 2007; Hunter, 2010; Nathan & Knuth, 2003). Their traditional classroom roles are challenged and they are pressed to attend to student reasoning in ways which differ from their usual practices (Mendez, Sherin, & Louis, 2007).

Teachers are influenced by their own experiences as students in classrooms. Given that most teachers experienced learning mathematics in traditional classrooms clearly their mathematical identity will be conflicted when asked to construct and teach within inquiry classrooms. Classrooms which have a diverse student community (for example, in New Zealand Māori and Pāsifika students and other minority groups) add to the challenge. We argue the need for support structures that allow teachers time to adjust their conceptual view of mathematics teaching and learning while guiding the construction of mathematical practices within an inquiry community. In this chapter we draw on a range of pedagogical practices which support the construction of inquiry classrooms through use of a *Communication and Participation Framework* (Hunter, 2008). This framework was designed to scaffold teachers to gradually transition students toward the use of more complex participation and discourse patterns which support them engaging in increasingly proficient mathematical practices. We present examples that show how teachers purposely drew on this framework as they changed their pedagogical practices and established the discourse patterns of inquiry and argumentation with diverse populations of students. Our aim is to provide models of how reconstruction of expectations and obligations for all students' participation in collaborative discourse can be occasioned in mathematics classrooms.

In the next section we examine the reasons why the space needs to be opened for all students to engage in mathematical practices. We look at the role of the teacher and students in mathematical inquiry classrooms. Most importantly we highlight the need for teachers to understand possible barriers their diverse and marginalised students may encounter and know ways in which they can overcome them.

MATHEMATICAL TALK AND MATHEMATICAL PRACTICES

The close connection between student engagement in mathematical talk and their mathematical achievement has been well documented. Staples (2008) draws on a cognitive perspective to describe how student engagement in collaborative discourse supports students to construct rich connected conceptual mathematical understandings. Inherent in this process are the opportunities they have to verbalise their reasoning and analyse and critique the reasoning of others. From a sociocultural perspective Staples (2008) explains how interacting in collaborative discourse allows students to engage in the core practices of mathematics—that is mathematical practices. Mathematical practices encompass a range of actions proficient problem solvers use; the mathematical skills and knowledge that constitutes expertise in learning and using mathematics. They include, for example, such practices as representing, inquiring, justifying and generalising reasoning (Boaler, 2003; RAND, 2003).

For too long mathematics education has focused predominantly on students constructing a body of mathematical knowledge. For students to develop robust reasoning processes, however, students need opportunities not only to construct a broad base of conceptual knowledge; they also require ways to build their understanding of mathematical practices (Hunter, 2013; RAND, 2003; Selling, 2016). A large body of empirical and theoretical research has illustrated that mathematical practices are essential in positively shaping students' mathematical disposition and competence to do mathematics (e.g., Boaler, 2003; Kazemi & Franke, 2004; Goos, 2004; Hufferd-Ackles, Fuson, & Sherin, 2004; Lampert, 2001; Staples & Truxaw, 2010). Selling (2016) proposes that for students to learn to engage in mathematical practices teachers need to be "responsive to students as they support collective participation in mathematical practices" (p. 513).

Our research, (Hunter, 2009; Leach, Hunter, & Hunter, 2014) also shows the need for teachers to respond 'in the moment' to productive student moves which support their engagement in mathematical practices. We also see the need, however, for direct attention to be given to the mathematical practices and the inquiry and argumentation discourse which supports its development, on the outcomes for diverse learners who may otherwise be marginalized in the mathematics classroom.

Teachers, as the more expert members of the community, take a significant role in constructing inclusive classrooms where respectful exchanges of ideas can occur. For teachers to achieve their students actively engaging in extended conversations which involve mathematical inquiry and challenge they need to negotiate the classroom learning context with the students and co-construct classroom, social and mathematical norms (Hunter, 2008; Sullivan, Zevenbergen, & Mousley, 2002; Weingrad, 1998). The social and mathematical norms refer to a network of obligations and expectations which influence and regulate the interactions in the classroom. Social norms provide members of the learning community with guidelines for acceptable ways to participate and communicate mathematical reasoning; mathematical norms relate specifically to the mathematics and being able to construct mathematically acceptable explanations, representations, justifications and generalisations.

Being required to work in an environment where the discourse extends to argumentation and more generalised reasoning is challenging for many students. We know from our research studies (e.g., Bills & Hunter, 2015; Leach et al., 2014) that when the diverse and often marginalised students (Māori and Pāsifika) we work with participate in collaborative discourse they learn a rich, connected mathematics. We recognize, however, that both the cognitive and participation demands on such students increase (Forman, Larreamendy-Joerns, & Brown, 1998; Staples & Truxaw, 2010). Teacher and student held beliefs and attitudes in regard to inquiry and argumentation need considering as well. These may be attributed to personal beliefs but they are also shaped by societal and cultural beliefs. For example, Andriessen (2006) explains how many students subscribe to a media-prevalent view of argumentation as oppositional behaviour; a form of negative behaviour which they consider not conducive for learning. Other studies (e.g., Bills & Hunter, 2015; Civil & Hunter, 2015; Hunter, 2007c; Hunter & Anthony, 2011) illustrate the cultural influences on students engaging in the discourse of inquiry and argumentation inherent in mathematical practices. These studies showed the dissonance students experienced when required to explain, justify, or inquire into the reasoning of others' because these practices were not common in their home environment or they perceived that these practices had potential to be interpreted differently within their community.

Clearly, constructing a sense of safety, trust and mutual respect is important if all students are to participate and contribute their mathematical reasoning. The teacher's role is critical in understanding challenges some students face and how to actively ensure all students' access to mathematical practices. Our own studies (Bills & Hunter, 2015; Civil & Hunter, 2015; Hunter, 2007c; Hunter & Anthony, 2011) and the studies of others (e.g., Lampert, 2001; White, 2003; Wood, Williams, & McNeal, 2006) show the importance of teachers ensuring student access to the mathematical content discussions and the social discourse process (when and how to explain, question, agree, disagree or challenge). The need for explicit teaching of 'politeness' strategies, the essential qualities or norms for ways students might question and disagree also need considering. Furthermore, our research with Māori and Pāsifika students highlights the importance of attending to, respecting, and building from their cultural norms and values as a way to shape their participation. Through these studies and a body of others, we know that without explicit discussion by teachers of the structure of collaborative discourse—how it works, its norms and rules—the spaces are not open for a significant group of students to participate fully and gain mathematically from the rich conversations of inquiry mathematics classrooms.

In the next section we look at the New Zealand context where this study is set. We look at the Māori and Pāsifika students who have the lowest mathematics achievement levels and experience the highest socioeconomics disparity of any ethnic groups in New Zealand. We describe a project that builds on the cultural capital of these students; a project which affirms their cultural norms and values and as a result provides them with space to access the mathematical talk and practices.

THE NEW ZEALAND CONTEXT

This chapter draws from a large project that involves over four hundred teachers in thirty two New Zealand Years 1–8 primary schools. The project began from a doctoral study (Hunter, 2007a) in which a group of teachers in an urban school in a high poverty area with predominantly Māori and Pāsifika students worked collaboratively with the researcher to develop a *Communication and Participation Framework* (Hunter, 2008). The Framework was a tool designed to scaffold teachers to engage students in reasoned mathematical practices within communities of mathematical inquiry. Subsequent iterations of the research gradually increased the number of schools, as the teacher educators and researchers (as mentors) deepened the focus on culturally responsive pedagogy and ambitious teaching (Kazemi, Franke, & Lampert, 2009). The members of staff from thirty-two schools, who are currently part of the project, attend four full days of professional learning over the year. Mentors also work with the individual teachers in their classrooms co-constructing mathematics together, on average once a month.

The students come from very low socio-economic home environments and are predominantly indigenous Māori or of Pācific nations heritage. This group of students has a long history of underachievement in mathematics in New Zealand classrooms (Bills & Hunter, 2015). The aim of the project is to work with schools and teachers to co-construct mathematical inquiry communities; learning communities in which the students are provided with multiple opportunities to engage in the discourse of inquiry and argumentation and a range of mathematical practices. At the core of the project is culturally responsive teaching (Gaye, 2010) and the need for teachers to attend to both the values and beliefs of the students and the cultural capital (Bourdieu, 1977) the students hold. This involves working with teachers to change the culture of the mathematics classroom so that it better matches the lives of the Māori and Pāsifika students. For example, particular focus is placed on the use and exploration of problematic and group worthy tasks within contexts relevant to the lived reality of Māori, Pāsifika, and other diverse learners (Bills & Hunter, 2015). The use of mixed ability grouping aligned with the cultural values of the students that support collectivism and communalism rather than individualism is examined (Hunter, 2007b).

We draw on the core Pāsifika values to underpin the social and mathematical norms constructed within the classroom learning community. These values include reciprocity, respect, service, inclusion, family, relationships, spirituality, leadership, collectivism, love, and belonging (Ministry of Education, 2013). Within a mathematics lesson the students are grouped heterogeneously and the Pāsifika concept of collectivism is used to structure the mathematics activity. The teachers draw student attention to the parallels in their cultural norms and model them when requiring students to ask and respond to questions and use mathematical argumentation in ways considered respectful as a Pāsifika person. The values also support students respecting the contributions of others in ways that break down status issues and improve equity. Drawing explicitly on the beliefs and values in culturally responsive ways supports the strengthening of prosocial skills, the construction of positive cultural identities and enhanced connections to not only mathematics but also to each other (Alton-Lee, Hunter, Sinnema, & Pulegatoa-Diggins, 2011).

Data collection annually has included teacher and student interviews, classroom artefacts, video captured observations of lessons, written and recorded teacher reflective statements and teacher recorded reflective analysis of video excerpts as well as student achievement data. In this chapter we have focused on the data across a group of teachers as they established the social and mathematical norms which supported mathematical practices within inquiry communities. Although this does carry the risk of over-generalising from selected teachers, the focus of this chapter is on the actions the teachers take to open the space for all students to engage in mathematical talk within collaborative inquiry and argumentation. The actions they take in the lessons are representative of the wider group of teachers within all the schools and they provide clarity around the types of actions teachers might take to establish learning communities that support all students to equitably access mathematical practices.

In this section we described the project and its core elements. In the next section we focus on the ways in which the different teachers established and maintained learning environments in which all students were able to participate and access the mathematical talk. We use examples of teacher talk and actions to show how teachers developed classroom environments. Of importance is the way in which the teachers drew on the actions outlined on the *Communication and Participation Framework* (see Appendix) to explicitly enact social and mathematical norms which supported the students to explain and justify their mathematical reasoning, and question and challenge each other.

PUTTING THE 'ACTIVE' INTO ACTIVE LISTENING FOR COLLABORATIVE SUPPORT AND SENSE-MAKING OF MATHEMATICAL EXPLANATIONS

Central to all students being able to participate in mathematical talk is their need to feel safe and supported by members of the classroom community. A key focus in the *Communication and Participation Framework* (CPF) is the establishment of both social and mathematical norms that support students being able to participate in discussions. Although mathematical practices overlap and interrelate, we contend that students need to be confident to explain and represent their mathematical reasoning before they are expected to justify and generalize it. An immediate starting point on the CPF of suggested teacher actions is for them to develop a supportive learning environment where the students feel confident to take risks when they interact, explain their reasoning, and inquire into the reasoning of others. Although these play out differently across the classrooms we consistently note that the teachers explicitly discuss changes in the mathematics lessons and the new roles they want the students to assume. The need for accountability for one's own listening is a clear focus as the teachers explore active listening as key to sense-making. For example, in the following excerpt the teacher is exploring the changed social norms inherent in the roles she wants the students to assume.

Teacher: *Okay so what is your job?*
Student 1: *Um…to listen.*
Teacher: *And how do you listen?*
Student 1: *By looking at them.*

The response from the student indicates that she holds a view common to many Pāsifika students. Pāsifika students often interpret their role in the mathematics classroom as a passive process of looking and listening not as an active process of sense-making where they are listening, looking and asking questions (albeit, sometimes silently in their heads). This links closely to the Pāsifika value of respect which may play out in a classroom where students interpret their role as needing to be respectful listeners particularly when interacting with older or more knowledgeable individuals. Realizing the ambivalence her Pāsifika students may regard towards their role in discussion the teacher continues to probe: *and how do you listen?* In reaction to the further press the student drops her head and remains silent. The teacher noting the student's response shifts her gaze to the wider group stating: *Hinemoa has taken a risk and now she needs somebody to help her out, please?* Within a Pāsifika context to not respond to an older person's question is considered ill-manners and so the dropped head indicated Pāsifika-mā [a loss of face]. However, the teacher's statement indicated to all the students that what Hinemoa had offered was a beginning point. To support the students to continue to contribute in ways that supported them both socially and culturally, rather than pressing her further she drew on the others. This indicated to the students that they were in a supportive environment where they were safe to take risks in joining a discussion but that it was not discourteous not to comply with the request to continue. Her continued push for more detail, however, also illustrated to the students that what they describe as active listening has many more layers. This showed that she recognized that, for many students, active listening needed deeper exploration to shift student beliefs beyond assuming that they can learn through merely listening and looking rather than as an active sense-making inquiry process.

Evident in all classrooms across our project is the close attention paid to developing active listening and engagement in the mathematical discourse. Core Pāsifika values within the students' home communities encourage them to learn by listening to their elders. The teachers in our project's classrooms discuss with the students the multiple voices they have which fit with the different roles they take. In the mathematics classrooms, active listening differs from the listening and voice they use in their home setting. As part of establishing social and mathematical norms which support a range of mathematical practices, the teachers discuss and role-play what active listening and inquiring looks like. The teachers build on the students' home listening practices but extend these to emphasize the need for listeners to sense-make and follow up with additional probing questions; questions which they would not use with their parents and elders in their home community. Building from the respectful home listening practices mathematical activity is shaped and extended to include collective engagement around constructing, questioning, and extending mathematical explanations into the school discourse of mathematical argumentation. Of importance is the emphasis placed on the collective activity of the students' sense-making together. For example, in one classroom a teacher began an activity by placing the students in a group of four and giving the following instructions:

> *I want you to talk to each other before you even touch the sticks. Lots of talking and listening, I might ask you what someone in your group said, not you, so you need to discuss things please and make sense of what someone else says.*

> *Listen carefully to each other and make sure you are asking questions. I want you to discuss what is happening in your patterns of two…and how many you have.*

Such actions provided the students with space to practise making explanations, questioning and responding to questions, all key elements of active listening. Not only had the teacher told the students that they were required to listen and question each other, she had also made them responsible for being able to explain their reasoning and the reasoning of the other members of the group.

The cultural values and ways of behaving in the home context of the students were also drawn on and used metaphorically to illustrate how the work of mathematics groups matched collective collaboration of people in their own cultural groups. The teachers matched cultural examples of how they wanted the groups to work together with the collective activities they knew the students engaged in their home context. Examples used included the concept of everyone rowing together in the same canoe (or waka or vaka; Māori or Pācific term for canoe). How the Pāsifika values of collective collaboration and reciprocity equated to the multidimensional collaboration required in the inquiry classroom is illustrated in the next example. A Sāmoan teacher began a lesson by discussing the making of siapo (the Samoan word for a fine cloth made from the bark of the Paper Mulberry tree). He led a discussion about how siapo is made by a group of people who collectively share their skills. In the discussion they agreed that siapo can only be made through the sharing of the collective skills. He then linked this discussion back to how the students needed to work together in their mathematics groups:

Teacher: *What are the benefits of a group working together?*
Student 1: *To make it faster and easier.*
Student 2: *To get the job done.*
Student 3: *You get confidence.*
Student 4: *You share ideas and ask questions.*

Although the students responded with ways that illustrated that they recognized the benefits of working together, the teacher wanted them to understand the need for multiple skills and expertise. He extended the discussion into a cultural context the children knew and understood from their home experiences:

Teacher: *When the women are making siapo do they all do the same job? No they don't because some do the beating, some do the drawings, different people have different skills…so someone who has the skill at rhythmic beating, someone who makes the dye that goes on it, and the person who draws the pictures, has the knowledge of the ancient patterns; that is a different person again. So it's the same in maths…if you are working in a group each person might have something different but you give together…what you share together makes a better group…So when you are working*

> *in a maths group…somebody is quite good at asking questions… somebody else might be good at saying let's draw a picture of what it looks like, somebody else might say actually I can see what you are drawing in that picture I am going to write it in numbers. So it is exactly the same as when you are making siapo…and if you do not work together then you miss out because your group fails. So everyone has a job to do and we share different ideas and skills to come up with our shared solution.*

Through his statement the teacher not only reinforced the need for both personal and collective responsibility but also the way in which all contributions were of equal value. He impressed upon the students the notion of multi-dimensionality; that all contributions were equally valid and valued. At the same time he showed the students the way in which their home values of communalism and collectivism transferred into their mathematical activity in school.

LIFTING THE CHALLENGE (AND RISK) TO CONSTRUCT MATHEMATICAL EXPLANATIONS AND QUESTION, AND RESPOND TO QUESTIONS ABOUT THEM

A teacher having high expectations for all students is significant to providing them with equitable opportunities to learn. Similarly, showing an ethic of care is important (Bartell, 2011; Noddings, 2005). Teachers who enact an ethic of care hold high expectations; they show they care enough to allow students to make mistakes, take risks, persist and overcome problems through effort and hard work. An ethic of care, however, can be misinterpreted by teachers and, rather than allowing students to struggle, they try to keep them safe using easily solvable problems and behaviors which allow the students to give up easily or respond in an inappropriate way. When the students show difficulties participating in the mathematical talk in appropriate ways some teachers misinterpret an ethic of care and decide that the behavior is a Pāsifika trait and allow it to continue. In the previous section, however, we described the confusion some Pāsifika students felt about how to actively listen. We described the different strategies teachers took to clarify active listening and engagement. Likewise, teachers need to use specific actions to scaffold students who have been limited by a lack of challenge and low expectations caused by well-meaning teachers who believed that they were doing the best for the students.

Ensuring an appropriate level of complexity and challenge in the problems is important for raising student achievement (Smith & Stein, 1998). In our project, as part of our culturally responsive pedagogy and to reduce the cognitive load for students, most often the tasks and problems are set within known cultural and social contexts. This provides the students with space to grapple with and understand difficult mathematics concepts and at the same time see mathematics as relevant to their lived contextual reality. For example, in a project classroom a teacher gave his students a proportional reasoning problem around the construction of an *Ule Lole* (a necklace used at Pāsifika celebrations made out of different sweets representing a *Lei*). When a student was questioned about how problems set within known contexts helped her, she stated:

Student: *It makes it easier for us to learn…like the Ule Lole (lolly necklace) problem because most of us have made it before and we can see it and have a picture in our minds so we can see how it's proportions and ratios like one chocolate to three Fruit Bursts or Minties.*

Teachers verbally preparing students for challenge in their mathematics are also a key element seen in many project classrooms. This signals to the students that the teacher knows that they will encounter challenge but that that is an expected part of doing mathematics. This is illustrated in a project classroom when a teacher gives the groups a problem and then acknowledges that he expects them to have to work hard to sense-make:

Teacher: *I know that this is a challenging problem and if you don't understand you've really got to help each other out and ask those questions… So it's quite a challenging problem but you're in groups, you should do well… I don't really want to see any quiet groups; I want to see you talking, questioning and really making sense of these problems.*

His statement showed the students his belief in their ability to persist and work as a collective and use inquiry as a tool to mathematically sense-make.

We see across the different project classrooms a focus placed on encouraging students to take intellectual risks in their mathematical reasoning. The teachers described the risks they were taking in changing how mathematics was being done in their classrooms. They, also, explicitly talked about the need for the students to take risks. For example in one classroom as the students explored equivalent fractions their teacher asked:

Teacher: *Is there anyone else who can model another equivalent fraction?*

When a student indicated their willingness to respond she affirmed the student saying:

Teacher: *Good Rata for taking a risk like this. Just go ahead and construct another fraction the same.*

Through her statement she explicitly identified that what Rata was doing was an example of intellectual risk-taking. Through such actions safe learning climates are constructed where students were able to take social and academic risks.

For teachers the risk in this changing mathematical environment is not insignificant. Many teachers in our project classroom shared with the students their own struggles with the shifts in pedagogy. For example, one teacher when talking to the students about how they would be challenged made a comparison with her own journey:

Teacher: *We are all going to be taking risks. That includes me; I am learning too, how to support you to talk more and I am struggling with not telling too much.*

Students who have experienced traditional mathematics instruction have learnt that '*teacher-talk'* dominates. They have learnt that their role is to listen rather than talk or question. As a result for many students learning to make mathematical explanations is difficult and needs direct addressing. In addition, many Pāsifika children have learnt to listen quietly rather than talk and ask questions, or disagree with others, which may hold the possibility of being considered rude and impolite (Civil & Hunter, 2015). In our project teachers grapple with ways they can ensure that they do not compromise the core Pāsifika home values of students while also providing them with a 'voice' appropriate to the mathematics classroom. In accord with actions outlined on the Communication and Participation Framework, the teachers provide models of ways in which they want their students to explain their mathematical reasoning. They, also explicitly show ways they can question, agree, disagree and challenge politely. In the following example, the teacher is explicitly scaffolding the students to provide a mathematical explanation:

Teacher: *You are going to explain how you are going to work it out to your group. They are going to listen. I want you to think about and explain what steps you are doing, each step you are doing, what maths thinking you are using. The others in the group need to listen carefully, and stop you and question anytime, or at any point where they can't track what you are saying.*

In this example the teacher has emphasized that the explanation needs to be structured so that the listeners can make sense of it, chunk by chunk. At the same time she wants the students to question the explanation step by step, in order to maintain their understanding of the mathematical reasoning as it is explained. Actions like these provided the students with space to rehearse their explaining and questioning in a way that maintained their confidence.

Close monitoring of the group members' sense-making is also evident in the classrooms. For example, in many classrooms the teachers sat alongside groups as they talked. Then they would halt the group at a specific point to provide space for questions. This is illustrated in the following example:

Teacher: *Is there someone in the group who has a question about a part or a bit of what Tipani was showing to us and explaining to us?*

Part of the close monitoring is noticing when the students use actions which support the use of mathematical practices. These need to be made explicit to the rest of the students so that they can see models of different actions which they can use to support their engagement in the mathematical talk and practices. This is illustrated in the next example:

Teacher: *Did you see that? For example as we are saying add two, that's what Ruru was saying, okay so we've got these two here. Then Alan asked a very good question, why aren't we adding on three each time? That is a great example of a question we can use to understand the explanation and make it clearer mathematically.*

Through the intervention she provided a model of an appropriate question students needed to ask to deepen their understanding of a mathematical explanation.

Developing the question-asking ability of students is of importance when working with our often marginalized Pāsifika students. Many students may not realize that asking questions is part of their individual accountability for sense-making. Traditional patterns of instruction support them to believe that it is the teacher's responsibility to ask questions. Building confidence to question is a lengthy process that requires teacher modeling, close watching, and listening. In accord with actions suggested on the *Communication and Participation Framework* developing the skill to provide conceptual explanations is matched with that of asking questions. A constant feature of our project's classrooms is the attention teachers give to scaffolding students to ask questions that match the different mathematical practices. For example, the teacher in the next example has specifically prompted her students to use questions that elicit further explanatory information:

Teacher: *Think about what he is doing. Think of the questions you need to ask, the 'what' questions.*

In other classrooms we see examples of teachers halting an explanation to explore with students the types of questions they need to ask in order to elicit further information:

Teacher: *What questions do they need to ask?*
Student: *I don't understand this. Could you please repeat it?*
Teacher: *Yes but if someone didn't understand it, though, and if the same thing was said to them…*
Student: *Explain it in a different way. Can you show me how you did that part and what you used?*

In this example the teacher has not only made obvious the limited outcomes that result when an explanation is repeated; she has pushed the students to question more widely at a deeper level. In the next example we observe a teacher monitoring student questioning of each other. She has judged that the question has not elicited the information the student required to make sense of the explanation and so she has halted the discussion and required that the student rethink and rephrase the question:

Student: *What have you done?*
Teacher: *No think again about your question and what you need to know about his thinking?*
Student: *What are you doing? Can you explain to us how that shows why you argue that Peter was wrong?*

In this example the teacher has caused the student to rephrase what he wants to know more clearly but at the same time she has indicated her trust that the student can persist with asking questions until his understanding is complete.

SHIFTING THE RISK BEYOND EXPLAINING TO ACTIVELY ENGAGING IN MATHEMATICAL ARGUMENTATION AND GENERALIZATION

Shifting the focus from providing mathematical explanation to justifying and engaging in mathematical argumentation is important if students are to generalize and develop rich conceptual understandings. Underpinning learning to justify is the need to learn to indicate agreement or disagreement with mathematical reasons and this poses many challenges to some students. Many students' view of disagreeing is shaped within a media frame where it is seen as a negative process. In addition, as we noted earlier being asked to engage in questioning and challenging may cause many Pāsifika students discomfort because within their home environment such behavior would be considered rude and ill-mannered. This requires direct attention to address need for Pāsifika students to be able to engage in an appropriate 'voice' for mathematics while also ensuring that they understand a need for a different 'voice' to use in different contexts including that used with Pāsifika elders.

Commonly, the teachers in our project address the need for students to engage in justification and argumentation directly. We see evidence of them discussing the different 'voices' the students use in the different situations including explaining, and justifying in mathematics. Other teachers talk about the need for them to engage in arguing, often terming it '*polite maths arguing'* so that the students can link their need to be polite and respectful in their home context with ways to act in the mathematics classroom. In the following example a teacher has introduced the concept of '*maths arguing'* through direct discussion with the students.

Teacher: *Arguing is not a bad word…sometimes I know that you think to argue is…I am talking about arguing in a good way. So please feel free if you do not agree with what someone has said. You can say that, as long as you say it in an okay way. A suggestion could be that you might say, I don't actually agree with you, could you show that to me. Do you think you could prove it mathematically, could you perhaps write it, or draw something to show that idea to me…and sometimes doing that the other person thinks it wasn't quite right so they change their idea and that's okay.*

Through the discussion the teacher has provided the students with a safe way to engage in argumentation. At the same time she has illustrated the benefits that emerge for both the explainer and questioner. The use of mathematical argumentation is also presented as a tool to support richer, more transparent reasoning and deeper conceptual understandings.

Teacher: *When you are doing your mathematical arguing it is because for some reason you don't agree with someone else's thinking, or perhaps the way someone else has done what they have done. Mathematicians engage in arguing, they do it all the time. It is good to have a healthy mathematical argument. You actually learn a lot more from arguing about your maths than not arguing about your maths. It just opens it all up…all the thinking.*

Through her actions she allowed the students to align their mathematical practices with those used by mathematicians.

To support students to accept disagreement and challenge as learning tools, the teachers on many occasions position the students to agree, or disagree with a conjecture. In the following example the teacher had prompted the students to compare two different explanations.

Teacher: *So do you think that's just a little bit quicker than your skip counting? Who agrees that that is a quicker way? Maybe you can say why. Who disagrees? You can disagree, people.*

The teacher, observing a child shake his head indicating his disagreement, directed him to provide further reasoning: '*Okay Manu just talk about it, what other ways can you use?*'

In other classrooms we see teachers providing space for students to practise and rehearse justifying their mathematical reasoning.

Teacher: *Okay put that down and now you are going to ask the group if they agree with that.*

At the nods of agreement the teacher then instructs the explainer to press further.

Teacher: *So ask them… How do you know that that equaled 24?*

In these examples the teachers have indicated to the students that they can take either position but that they needed to be able to validate and explain the reasoning for the position they took.

In many classrooms we also observe teachers closely monitoring for individual students who actively support group members to challenge thinking or to promote argumentation. Their actions are then used as exemplars of good practice for the other students. For example the teacher in the following excerpt ended the group work with an observation of a student's behavior she wanted the other students to model on.

Teacher: *There's lots of really interesting korero [talk] going on. I really spent most of my time with this group listening to them because they were having problems and arguments and Wiremu was really good…you were really good in that position Wiremu, you were helping your group and you weren't giving out the answers and that's really good but you were really pushing them to think. Yes you had everyone talking about and discussing how they were going to sort out the ideas. You were challenging and other people were following your lead so the arguing was really kapai [good].*

Her talk made visible the kinds of behavior that supported the use of mathematical practices. She highlighted her role as a listener while illustrating for the students that solution-finding through questioning, struggling, formulating and reformulating thinking on the way to constructing conceptual understandings was a normal part of doing mathematics. This teacher—like so many teachers in our project—had opened the space and provided the students with an environment where they were confident to use a range of mathematical practices within the discourse of inquiry and argumentation.

DISCUSSION AND CONCLUSION

We opened this chapter with a discussion about the way in which mathematical talk and who gets to participate in it has a significant effect on who is able to learn mathematics with rich conceptual understandings. We argued that all students need opportunities to engage in, and build their understandings of a range of mathematical practices as a key equity issue and we noted the significant role teachers have in supporting students to achieve this. We recognized, however, the many challenges that teachers and students have in doing this. Teachers have to move past their own previous history as learners and teachers in traditional mathematics classrooms where '*teacher-talk*' dominates and this involves risks for teachers too. As a number of researchers (e.g., Hodge, 2008; Sherin, 2002; Wood, 1999) have shown, opening the dialogue and supporting student led reasoning adds another level of complexity. This complexity is underpinned by some teacher actions that, on the surface, appear simple. Take active listening, for example, which many teachers assume all students understand; in our project classrooms we encounter the opposite. It is the explicit and on-going attention to active listening, which supports the students to realize their need to self-monitor their understandings. Other mathematical practices are learnt through teachers scaffolding students to engage in them while participating in mathematics; while others are learnt through teachers drawing student attention to them and making them explicit when used.

The classroom climate is critical to the learners within it. Like Weingrad (1998) and White (2003) we have shown that for diverse students (like the Pāsifika students in our project classrooms) to engage in productive discourse requires a consistent and continual focus on the enactment of a respectful classroom climate in which student contribution is valued and built on. Building both a classroom environment and student confidence to participate in mathematical practices is a lengthy and patient enterprise. The social norms of the learning environment are central to all students having opportunities to participate in a range of mathematical practices. In our classrooms we extend the work of a teacher to develop a respectful learning environment that intentionally builds on the lived cultural and social values of the students through deliberate pedagogical actions. As Weingrad (1998) showed, intellectual risk-taking requires an environment that is respectful of student reasoning. In addition, for our Pāsifika students as novices in an environment of inquiry and challenge, explaining and answering questions about the mathematical thinking potentially poses considerable risks to their self-esteem and their sense of cultural self-worth. Learning to explain mathematical thinking and then be able to question and respond to questions takes practice as does justifying and engaging in mathematical argumentation. At the same time, in this chapter we have shown the importance for our Pāsifika students to be supported to do this in 'polite' and respectful ways and which allow them to maintain their mana (self-esteem).

In this chapter we have shown that teacher actions which focus on simultaneously engaging students in discussion of mathematical content and discussion on how to engage in mathematical discussion has the potential to deepen their participation in intellectual risk taking and argumentation. Through the many examples we have offered in this chapter and the *Communication and Participation Framework* we have provided a way forward for teachers consider as they construct learning environments in which all students have opportunities to access the different mathematical practices.

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APPENDIX

Communication and Participation Framework: Teacher Actions to Engage Students in Mathematical Practices

Developing conceptual explanations including using the problem context to make explanation experientially real			
Model providing a mathematical explanation. Use the context of the problem not just the numbers.	Re-voice, and extend an explanation using the problem context. Expect mathematical reasons not "tidying" 19+7=20+6 because 6+1=7 and 19+1=20	Question to scaffold students to extend their explanations to include the problem context and what they did to the numbers mathematically.	Model and support the use of questions which clarify an explanation. What do you mean by? What did you do in that bit? Can you show us what you mean by? Could you draw a picture of what you are thinking?

Have the students develop two or more ways to explain a strategy solution which may include using materials	Compare explanations and develop the norm of what makes an acceptable explanation. Reinforce what makes it mathematical	Launch the problem and have the students read it as a group, discuss, interpret, reinterpret collectively using student voice.	Shortly after the small groups begin to solve a problem as a large group have them describe their different starting point. Reinforce acceptability of multiple ways. Support them to make connections to other or previous problems.	
Ask the students in small groups to examine their explanations and explore ways to revise, extend and elaborate on sections they think others might not understand			Have students examine their explanation, predict the questions they will be asked and prepare explanations.	
Active listening and questioning for sense-making of a mathematical explanation				
Discuss and role-play active listening. Use inclusive language “show us”, “we want to know”, “tell us”.	Structure the students explaining and sense making section by section.	Emphasise need for individual responsibility for sense-making	Provide space in explanations for thinking and questioning	Affirm models of students actively engaged and questioning to clarify sections or gain further information
Collaborative support and responsibility for the reasoning of all group members: use core Pasifika values				
Provide students with problem and think-time then discussion and sharing before recording	Establish use of one piece of paper one pen. Explore family structures where everyone participates	Establish expectation that students agree on construction of a solution strategy that all members can explain.	Explore ways to support students indicating need to ask a question during large group sharing. Use no hands up or the use of koosh balls, or pegs, or beany toys	
Explore ways for the students to support each other using a range of cultural models e.g. all in the same waka paddling together, or a kapa haka group which requires the expert to be responsible to bring the group up to their level of expertise		Select a different member of the small group to explain than the recorder	During large group sharing change the explainer mid explanation	When questions are asked of the small group select different members to respond (not the recorder or explainer)

Developing justification and mathematical argumentation				
Require that students indicate agreement or disagreement with part of an explanation or a whole explanation	Ask the students to provide mathematical reasons for agreeing or disagreeing with an explanation. Vary when this is required so that the students consider situations when the answer is either right or wrong.		Model and support the use of questions which lead to justification like 'How do you know it works?', 'Can you convince us', 'Why would that tell you to', 'why does that work like that', 'so what happens if you go like that', 'are you sure it's', 'so what happens if', 'what about if you say…does that still work', 'so if we	
Ask the students to be prepared to justify sections of their solution strategy in response to questions.	Require that the students analyse their explanations and prepare collaborative responses to sections they are going to need to justify	Model ways to justify an explanation "I know 3 + 4 = 7 because 3 + 3 = 6 and one more is 7".	Structure activity which strengthens student ability to respond to challenge	Encourage the use of 'so if', 'then', 'because' to make justifi-cations. Use this format to validate an explanation
Expect that group members will support each other when explaining and justifying to a larger group	Explicitly use wait time or think time before requiring students to respond to questions or challenge	Require that the students prepare ways to re-explain in a different way an explanation to justify it.	Provide wait time to allow students to prepare questions which lead to justification	

Developing representing as part of exploring and making connections (How can I/we make sense of this for my/ourselves). Communication and justification (How can I explain, show, convince other people)			
Expect the use of a range of representations including acting it out, drawing a picture or diagram, visualising, making a model, using symbols, verbalising or putting into words, using materials.	Expect the students to explain and justify using the representation as actions on quantities not manipulation of symbols (use context)	Require that the students compare and contrast representations and evaluate for efficiency	Ask students to re-represent their thinking in different forms in response to questions or for clarification

Developing the use of mathematical language

Expect the use of mathematical language to describe actions while making mathematical explanations	Expect the use of correct mathematical terms. Ask questions to clarify terms and actions on symbols (using the context).	Require the use of mathematical words to describe actions. Reword or re-explain mathematical terms and mathematical explanations. Use other examples to illustrate meaning.	Require students that the students pose questions using appropriate mathematical language.

Developing generalisations: Representing a mathematical relationship in more general terms. Looking for rules and relationships. Connecting, extending, reconciling

Ask the students to consider what steps they are doing over and over again and begin to make predictions about what is changing and what is staying the same.	Ask the students to consider if the rule or solution strategy they have used will work for other numbers. Consider if they can use the same process for a more general case. (e.g., what happens if you multiply any number by 2)	Model and support the use of questions which lead to generalisations like 'Does it always work?', Can you make connections between', ' Can you see any patterns? Can you make connections between', How is this the same or different to what we did before', Would that work with all numbers',